The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so we are led to the more general conclusion that the mass of a body is a measure of its energy content.

The perplexing question we are left with is why energy radiation is needed to explain $E=Mc^2$ but energy radiation is expressly forbidden if we are to correlate $E=Mc^2$ with the formula for relativistic mass increase. Let us resolve this problem by showing how we can deduce $E=Mc^2$ from the very opposite viewpoint, the fact that there is no radiation of energy by accelerated charge.

The Energy-Mass Formula

Consider the problematic Larmor formula:

$$\frac{dE}{dt}=\frac{2e^2f^2}{3c^3} \tag{73}$$

and examine its derivation. It expresses the rate at which energy is radiated from an electric charge e when accelerated at the rate f.

The formula was founded upon the assumption that waves are developed by an accelerated charge and spread remote from the charge into empty space. Then, by the additional assumption that energy is carried by these waves, an energy radiation as given by the Larmor formula is obtained. The effects of the accelerating field are irrelevant at large distances and do not affect the waves. Accordingly, the accelerating field need not be considered in the classical derivation of the formula. It is this latter comment that attracts our attention in this critical examination.

Let us first summarize how the Larmor formula is derived using a textbook method attributed to J. J. Thomson. Refer to Fig. 21. At a point P in the wave zone distant ct from a charge e centred at O the electric field disturbance which gives the energy radiation is of the form:

$$\frac{ef\sin\theta}{c^3t} \tag{74}$$

Here θ is the angle between OP and the direction of an accelerating electric field V or acceleration f. The field given by (74) is at right angles to the electric field of e acting along OP.

The Larmor formula is deduced by integrating the energy density attributable to this field term (74) for an elemental volume $2\pi(ct)^2 \sin\theta \, cdt \, d\theta$ between the limits $\theta=0$ and $\theta=\pi$, and then doubling the result to allow for the equal contribution of magnetic field energy

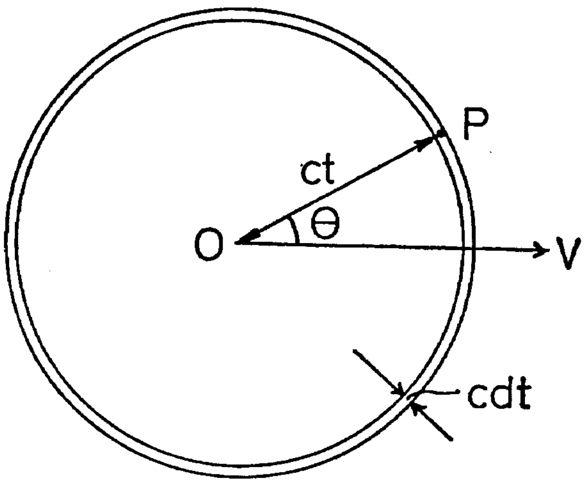

Fig. 21

and electric field energy characteristic of Maxwell's equations. This will give the energy radiated in the time interval dt. The result is:

$$2\int_0^{\pi}\left[\frac{1}{8\pi}(ef\sin\theta/c^3t)^2 2\pi(ct)^2\sin\theta \, cdt\right]d\theta = 2e^2f^2dt/3c^3 \qquad (75)$$

At this stage we are not interested in what happens remote from the charge. We question the assumption that energy is radiated at all and concentrate attention on the source of the alleged radiation. This is where the accelerating field V does its work and interacts with the field set up by e itself. The field energy density must then include the interaction with V omitted from the derivation of the Larmor formula. The field given by (74) is then:

$$\frac{ef\sin\theta}{c^3t} - V\sin\theta \qquad (76)$$

Squaring this and restricting attention to the time-dependent components, we obtain:

$$(ef\sin\theta/c^3t)^2(1 - 2c^3tV/ef) \qquad (77)$$

It is then immediately evident that there is no energy radiation if the latter part of this expression is zero, that is, if:

$$Ve/f = e^2/2c^2(ct) \tag{78}$$

Since it is the basic hypothesis of this attempt to deduce $E = Mc^2$ that there is no radiation of energy, we must admit (78). To proceed, let us distinguish between an electric charge confined by a boundary of radius a and the empty space surrounding this charge boundary. Regard the field of the charge in this surrounding space as an integral system. On this basis we may expect the Coulomb self-energy of the electric charge in the field surrounding the charge to exhibit a single-valued mass property related to the energy:

$$E = e^2/2a \tag{79}$$

E is now energy associated with the charge e but located outside radius a. This is the energy corresponding with the expression Ve/f in (78) when ct is equal to a. Therefore:

$$Ve/f = E/c^2 \tag{80}$$

becomes the condition for no energy radiation across the radius bounding the charge. Ve/f then becomes the mass property associated with the Coulomb energy E. We have arrived at the anticipated result that $E = Mc^2$.

We must now consider the case in which the charge e is so distributed within the sphere of radius a that there is additional Coulomb energy within this sphere. We will adhere to the assumption that the self-energy of any charge exhibits a single-valued effect outside the spherical boundary confining that charge. In line with this the mutual interaction Coulomb energy of two spherical shell elements of the same body of charge will be deemed single-valued outside the outermost shell. It is, of course, zero within this shell.

In the case to be considered we regard the whole body of charge in uniform acceleration f. Thus a whole succession of shells of charge de_x of thickness dx at radius x undergo acceleration at the rate f simultaneously. It may then be shown, by tracing through the above analysis and developing a formula such as (75) based upon (76) rather than (74), that the energy radiated in time dt is given by:

$$(4f^2/3c^2)[\sum de_x(\sum de_x/2c^2 - Vct/f)] \quad (81$$

where the value of ct is equal to the higher x value for any cross-product component term involving $de_x de_x$. The reason for this is evident if we write the two terms as de_x and de_y, where y is greater than x. For $(de_x)^2$ there is no radiation from the radius $ct = x$, the actual radius of the charge de_x. Similarly for $(de_y)^2$ there is no radiation of energy from the radius $ct = y$. For $(de_x)(de_y)$ there is no energy within the radius y and we can only look for radiation from the radius y, but as we say there is none then the condition that (81) is zero is that the value of $ct = y$ applies to cross-product terms at the higher charge radius.

For each such component interaction it is then evident that the identity:

$$\frac{(de_y)\sum(de_x)}{2c^2} = \frac{(de_y)Vy}{f} \tag{82}$$

with y greater than x applies and may be written in the form:

$$\frac{(de_y)\sum(de_x)}{2y} = \frac{V(de_y)c^2}{f} \tag{83}$$

Bearing in mind that all charge interactions are counted twice in a summation, the left-hand side of the above expression is the Coulomb interaction energy component dE. Summing this for the total charge e gives:

$$E = (Ve/f)c^2 \tag{84}$$

This is the same as (80) but it now applies generally to any spherically-symmetrical charge distribution confined within a bounding sphere. It tells us that such a body of charge will, when subject to the field of other charge, be bound to move with an acceleration f if it is to avoid dispersing its energy by radiation. Thus we have deduced the property of inertia. By denoting Ve/f as the mass M we obtain:

$$E = Mc^2 \tag{85}$$

By the above analysis it is seen that there is a very good case for developing the $E = Mc^2$ formula based on the assumption that energy is not radiated. The flaw in the Larmor formulation has been discovered. It did not take account of the effects in the near vicinity of the charge due to the interaction of the applied accelerating field. However, all we have shown is that no energy emerges from a discrete charge when accelerated. This does not mean that the collective actions of many charges and the propagation of electromagnetic waves by accelerated charges play no role in energy transfer. Nevertheless one does need to be cautious about the assumptions in conventional field theory that energy is radiated, bearing in mind the scope for energy fluctuations within the sea of energy which appears to pervade space.

The essential point made in the above analysis is that the mass property is related exclusively to the intrinsic Coulomb energy of the discrete charge. This raises the question of how this energy is augmented when the charge is accelerated to increase the mass. Is the charge compacted into a smaller volume? Alternatively, are we to expect perhaps the creation of charge pairs in some quantum statistical manner? At least from the analysis in Chapter 2 we know that we need not look also for separate explanation of magnetic energy. This is a reacting kinetic energy and so must be a Coulomb energy associated with reacting charge.

Charge Equivalence

It is important to note that the derivation of the $E=Mc^2$ formula developed above involves a parameter c which is not the assumed electromagnetic propagation speed, but rather the speed at which an electric field disturbance propagates from an electric charge. This distinction commands attention because the parameter c for electric disturbance could well be more fundamental than that for electromagnetic disturbance. This leads us to consider the problem of charge equivalence, that is the identity of electric charge and that in evidence in electromagnetic actions.

First we consider the Principle of Equivalence given such great attention in Einstein's theory. This is the identity of inertial and gravitational mass. This is one of the earliest known facts of experimental physics. Galileo's legendary experiment at the leaning tower of Pisa and the later experiment in 1891 by Eotvos confirmed this equivalence. Further experiments by Dicke* have checked the accuracy of this equivalence to less than one part in 10^{10}.

Einstein's theory elaborates on the theme of equivalence of inertial and gravitational acceleration but it takes us no nearer to an understanding of the physical basis of the constant of gravitation G. Nor is there anything particularly surprising about the discovery that the mass which we know from inertia happens to be the mass developing

* R. H. Dicke, *Scientific American*, **205**, 84 (1961).