$$\frac{1}{4\pi}(H_{1z}H_{2z}) \qquad (10)$$

As was discussed in Chapter 2, mutual magnetic energy is equal and of opposite magnitude to the mutual dynamic electric field energy. Indeed, the two sum to zero. Electric field energy has mass properties. This follows from the discussion of the velocity-dependence of mass in Chapter 1. We need not think in terms of the motion of magnetic energy. Consequently, in considering the motion of energy and its mass properties, expression (10) represents the energy density of the electric field which has to move from $P$ to $Q$ as the wave passes through these points. This is a measure of the mass *redistribution* in the field. The main energy terms, that is the non-interaction terms, are related to the self energies of the moving charges. The faster they move, the greater their dynamic electric field energies. Hence, the greater their masses, as explained in Chapter 1. Interaction itself does not augment mass in the system shown in Fig. 1. Interaction means repositioning of mass. The passage of the wave can result in angular momentum being imparted to the field energy.

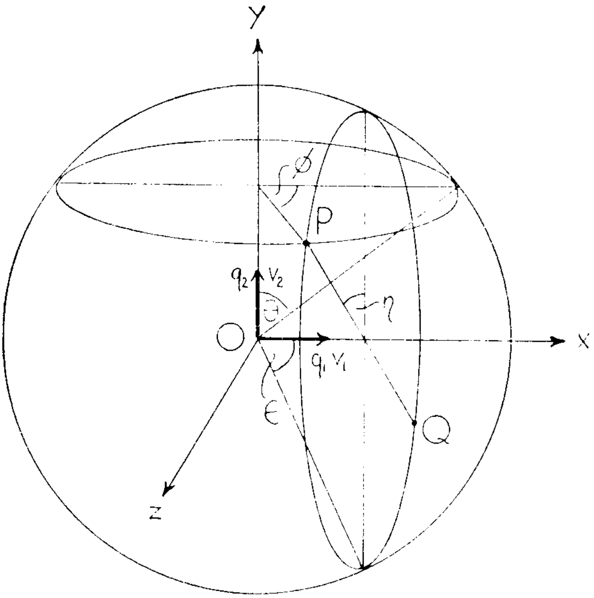

Fig. 1

To calculate the angular momentum of this field reaction we note that mass is moving around the wave region about the axis $Oz$.

Movement from $P$ to $Q$ is through an arc subtending the angle $2\varepsilon$ at radius $OP$ but projected by multiplication by $\cos\eta$. This movement is completed in the time taken for the wave to cross the region contributing to the energy interchange. Let $w$ be the angular velocity of the energy transfer. Then the projected velocity moment is $w(OP)^2\cos\eta$, and, since $w$ is $2\varepsilon/\mathrm{dt}$, where $dt$ is the time taken by the transfer, this velocity moment is:

$$2\varepsilon(OP)^2\cos\eta/dt \tag{11}$$

The radial thickness of the region under study is $cdt$ and an elemental volume at $P$ or $Q$ can be formed by multiplying $cdt$ by $2\pi(OP)\sin\varepsilon$ and $(OP)d\varepsilon$. Thus, the elemental energy being transferred between these volumes at $P$ and $Q$ is found, from (10), as:

$$\tfrac{1}{2}(OP)^2 cdt(H_{1z}H_{2z})\sin\varepsilon\, d\varepsilon \tag{12}$$

We divide this by $c^2$ to obtain mass and multiply the result by (11) to determine the angular momentum as:

$$\frac{1}{c}(H_{1z}H_{2z})(OP)^4\varepsilon\sin\varepsilon\cos\eta\, d\varepsilon \tag{13}$$

Substituting now the originally stated values of $H_{1z}$ and $H_{2z}$ gives:

$$(q_1q_2v_1v_2/c^3)\varepsilon\sin^2\varepsilon\sin\theta\cos\varphi\cos^2\eta\, d\varepsilon \tag{14}$$

It may be seen from Fig. 1 that:

$$\cos\varepsilon=\sin\theta\cos\varphi \tag{15}$$

From (14) and (15) the elemental field angular momentum given by (14) is obtained in terms of $\varepsilon$ and $\eta$. When averaged for all values of $\eta$, $\cos^2\eta$ becomes $\tfrac{1}{2}$. Thus the total angular momentum may be found by evaluating:

$$\tfrac{1}{2}(q_1q_2v_1v_2/c^3)\int_0^{\pi/2}\varepsilon\sin^2\varepsilon\cos\varepsilon\, d\varepsilon \tag{16}$$

This is:

$$\left(\frac{\pi}{12}-\frac{1}{9}\right)q_1q_2v_1v_2/c^3 \tag{17}$$

Consideration shows that if $v_1$ and $v_2$ are not at right angles, as shown in Fig. 1, the expression has to be multiplied by the sine of the angle between them. Thus (17) is a measure of the maximum angular reaction between the charges.